A New Result in the Synthesis of Incommensurate Line Networks

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Abstract: In this paper, an approach to computing the scattering matrix of a two-dimensional lossless network with two elements is first presented. Then, three examples are given to show how an incommensurate line network is realized. After that, recurrence formalas for calculating the scattering parameters of n cascaded two-dimensional networks are derived. Finally, to exhibit the outstanding merits of the new approach, a broadband double matching incommensurate line network is synthesized.

1 Introduction

In recent years, important results have been published treating systems with commensurate lines [1-2]. However, few papers have dealt with the synthesis of incommensurate line networks, except for the paper written by Kinariwala [3]. Though he has shown that an exponential transformation can often lead to a more efficient synthesis of a transfer function realizable by cascaded incommensurate lines, no method was provided to achieve the physically realizable transfer function. This problem existed also in the papers [4-6] for handling networks having both lumped elements and commensurate lines. So far many elegant theoretical results have been obtained but as yet no practical embodiment of these ideas is available for design purpose. The major difficulty is that no general method has been found to achieve a physically realizable multivariable transfer function. In order to attach this problem, we consider first the scattering parameters of a two-dimensional lossless 2-port network with only two elements, because one can always divide a cascaded multivariable network into several subnetworks, each of them having two complex frequency variables and containing only two elements. By investigation, we found that general expressions for the physically realizable scattering parameters of such subnetwork can be achieved in terms of the unitary property of scattering matrix of a lossless network. Therefore, a typical class of cascaded incommensurate line networks having $2n$ independent complex frequency variables $\lambda_i(i = 1, 2, \cdots, 2n)$ can be realized by synthesizing the scattering parameters of each subnetwork.

In this paper, the relations between the coefficients of numerator and denominator of scattering parameters of a two-dimensional lossless network having only two elements are first presented. Then, several examples are given to show how the network is realized. After that, recurrence formulas for calculating the scattering parameters of an incommensurate line network built by n subnetworks in cascade are derived. To exhibit the outstanding merits of the new method, a broadband double matching incommensurate line network is finally synthesized.

2 The scattering matrix of a two-dimensional lossless network with two elements

First assume that a 2-port network N is passive, lossless, and reciprocal. Then, as a natural extension of theorem 1 mentioned in [2], the following theorem for two-variable bounded scattering matrix is given by referring to [6].

Theorem 1: The scattering matrix $S(\lambda_1, \lambda_2)$ represents a lossless structure in both λ_1 and λ_2 domain if and only if

1. $S(\lambda_1, \lambda_2)$ is bounded real when λ_1 and λ_2 are real.
2. $S(\lambda_1, \lambda_2)$ is holomorphic in $Re(\lambda_1) > 0$ and $Re(\lambda_2) > 0$. (Except for a possible branch point pair of order 2 at $\lambda_1 = \pm 1$ and/or $\lambda_2 = \pm 1$, i.e., a factor $\sqrt{1-\lambda_1^2}$ and/or $\sqrt{1-\lambda_2^2}$.
3. $I - S(\lambda_1, \lambda_2)\tilde{S}(\lambda_1, \lambda_2)$ is nonnegative definite $Re(\lambda_1) > 0$ and $Re(\lambda_2) > 0$.
4. $S(\lambda_1, \lambda_2)$ is paraunity almost everywhere on $Re(\lambda_1) = 0$ and $Re(\lambda_2) = 0$. Thus $S(j\Omega_1, j\Omega_2)S^*(j\Omega_1, j\Omega_2) = I$ or $S(\lambda_1, \lambda_2)S^T(-\lambda_1, -\lambda_2) = I$ where $\Omega_1 = Im(\lambda_1), \Omega_2 = Im(\lambda_2)$ are the imaginary parts of λ_1 and λ_2, respectively. $(\ )^*$ means complex conjugate, $(\ )^T$, transpose, and $(\tilde{\ }) = ((\ )^*)^T$. I is the identity matrix.

Now we consider a network N having only two elements and being of two-dimensions, i.e., one element corresponding to λ_1 and the other to λ_2. Then, its scattering parameters, $s_{ij}(\lambda_1, \lambda_2)$, satisfying the condition of Theorem 1 can be written in general as

$$s_{11}(\lambda_1, \lambda_2) = \frac{h(\lambda_1, \lambda_2)}{g(\lambda_1, \lambda_2)} = \frac{h_1 + h_2\lambda_1 + h_3\lambda_2 + h_4\lambda_1\lambda_2}{g_1 + g_2\lambda_1 + g_3\lambda_2 + g_4\lambda_1\lambda_2}, \qquad (1a)$$

$$s_{12}(\lambda_1, \lambda_2) = s_{21}(\lambda_1, \lambda_2) = \frac{f(\lambda_1, \lambda_2)}{g(\lambda_1, \lambda_2)}, \qquad (1b)$$

$$s_{22}(\lambda_1, \lambda_2) = -\frac{f}{f^*} \times \frac{h^*}{g}, \qquad (1c)$$

where $f^* = f(-\lambda_1, -\lambda_2)$; $h^* = h(-\lambda_1, -\lambda_2)$; h_i and $g_i(i = 1, 2, 3, 4)$ are the coefficients of numerator and denominator polynomials, $h(\lambda_1, \lambda_2)$ and $g(\lambda_1, \lambda_2)$, of s_{11}; $f(\lambda_1, \lambda_2)$ are the numerator of s_{12}. Let's consider the following four cases:

(a) The network N comprises a length of transmission line (simply called *unit element* (UE) and a low-pass element, such as a series short-circuited stub (or inductance), or a shunt open-circuited stub (or capacitance). In this case, $f(\lambda_1, \lambda_2)$ can be represented as

$$f(\lambda_1, \lambda_2) = \sqrt{1 - \lambda_2^2}, \qquad (2)$$

where $\lambda_i(i = 1, 2)$ are defined as Richards' variables [1] for distributed elements or as normal frequency variables for lumped elements, and the UE is assumed to be the function of λ_2. Thus, in terms of paraunity property of scattering matrix of a lossless 2-port, i.e.

$$s_{11}(\lambda_1, \lambda_2)s_{11}(-\lambda_1, -\lambda_2) + s_{12}(\lambda_1, \lambda_2)s_{12}(-\lambda_1, -\lambda_2) = 1, \qquad (3)$$

the relations between h_i and g_i can be found as shown below

$$g_1 = 1, \quad g_2 = |h_2|, \quad g_3 = \sqrt{1 + h_3^2}, \quad g_4 = |h_4|. \qquad (4)$$

Note that $h_1 = 0$ holds for lowpass network having no ideal transformer. If h_2 and h_4 are known, h_3 will be fixed and can be computed from h_2 and h_4 via

$$h_3 = \frac{h_2^2 - h_4^2}{2h_2|h_4|}. \qquad (5)$$

(b) The network N comprises two UEs. In such a case, one has

$$f(\lambda_1, \lambda_2) = \sqrt{1 - \lambda_1^2} \times \sqrt{1 - \lambda_2^2}, \qquad (6)$$

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where both UEs are the functions of λ_1 and λ_2, respectively. Similarly, one can deduce g_i from h_i via

$$g_1 = 1, \quad g_2 = \sqrt{1+h_2^2}, \quad g_3 = \sqrt{1+h_3^2}, \quad g_4 = \sqrt{1+h_4^2}, \quad (7)$$

in which $h_1 = 0$ because of the situation similar to the case (a) and

$$h_3 = h_2 g_4 + g_2 |h_4| \quad (8a)$$

or

$$h_3 = h_2 g_4 - g_2 |h_4|. \quad (8b)$$

It can be seen that when h_2 and h_4 are specified, h_3 will be determined by (8a) or (8b).

(c) The network N comprises a UE and a high-pass element, such as a series open-circuited stub (or capacitance), or a shunt short-circuited stub (or inductance). Then, $f(\lambda_1, \lambda_2)$ is specified by

$$f(\lambda_1, \lambda_2) = \lambda_1 \sqrt{1-\lambda_1^2}. \quad (9)$$

Under these assumptions, g_i can be derived in terms of (3) as the functions of h_i, i.e.

$$g_1 = |h_1|, \quad g_2 = 1, \quad g_3 = |h_3|, \quad g_4 = \sqrt{1+h_4^2}, \quad (10)$$

where $h_2 = 0$ due to the situation similar to the case (a) and h_4 is related with h_1 and h_3 by

$$h_4 = \frac{h_1^2 - h_3^2}{2h_1|h_3|}. \quad (11)$$

(d) The network N comprises only two types of elements, i.e. short- and open-circuited stubs (or inductance and capacitance). One is related to λ_1 and the other to λ_2. In this case, *lossy transformation technique* (LTT) [7] can be employed to achieve the unit normalized scattering matrix, S, of the network N from the corresponding unit normalized scattering matrix, $\tilde{S}$, of a lumped lossless reference network M and vice-versa. That is,

$$S(\lambda_1,\lambda_2) = [\sqrt{Z_1 Z_2}(I+\tilde{S})-(I-\tilde{S})][\sqrt{Z_1 Z_2}(I+\tilde{S})+(I-\tilde{S})]^{-1}, \quad (12a)$$

$$\tilde{S}(\lambda) = [(I+S) - \sqrt{Z_1 Z_2}(I-S)][(I+S)+\sqrt{Z_1 Z_2}(I-S)]^{-1}. \quad (12b)$$

In (12b), $\lambda = \sqrt{Z_1/Z_2}$ and I is the identity matrix. The entries of $\tilde{S}(\lambda)$ are supposed to be in the form

$$\tilde{s}_{11}(\lambda) = \frac{a(\lambda)}{b(\lambda)} = \frac{a_1 + a_2\lambda + a_3\lambda^2 + \cdots + a_{n+1}\lambda^n}{b_1 + b_2\lambda + b_3\lambda^2 + \cdots + b_{n+1}\lambda^n}, \quad (13a)$$

$$\tilde{s}_{12}(\lambda) = \tilde{s}_{21}(\lambda) = c(\lambda)/b(\lambda) = \lambda^k/b(\lambda), \quad (13b)$$

$$\tilde{s}_{22}(\lambda) = (-1)^{k+1} a(-\lambda)/b(\lambda), \quad (13c)$$

where n specifies the maximum number of reactive elements in M. a_i and $b_i (i = 1, 2, \cdots, n+1)$ are the coefficients of numerator and denominator polynomials, $a(\lambda)$ and $b(\lambda)$, of $\tilde{s}_{11}$. The numerator polynomial, $c(\lambda)$, of $\tilde{s}_{12}$ is simply represented by λ^k. It implies that the network M is assumed to have k high-pass elements and $n-k$ lowpass elements.

Also, Z_1 and Z_2 in (12) represent the frequency-dependent parts of impedances of two types of the network elements, respectively. Their impedances can be simply written in the form

$$z_1 = r_i Z_1 \quad \text{and} \quad z_2 = r_i Z_2, \quad (14a, 14b)$$

where r_i is real positive multiplicative constant. In this case, four corresponding combinations for Z_1 and Z_2 can be considered, i.e.

a) $Z_1 = \lambda_1, \quad Z_2 = \lambda_2$ (15a)
b) $Z_1 = \lambda_1, \quad Z_2 = 1/\lambda_2$ (15b)
c) $Z_1 = \lambda_2, \quad Z_2 = 1/\lambda_1$ (15c)
d) $Z_1 = 1/\lambda_2, \quad Z_2 = 1/\lambda_1$ (15d)

Note that although in this section we only consider the network M(i.e. N) having 2 elements($n = 2$), n can be any integer! It means that the network N can comprise any large number of the two types of the elements. Furthermore, from (15), we know that four different types of the network N can be synthesized from $S(\lambda_1, \lambda_2)$ by replacing Z_1 and Z_2 in (12a) by one of the four combinations for Z_1 and Z_2, respectively. Finally, in comparison with the method developed by Rhodes and Marston [8], it is apparent that the LTT is considerably simplified and easy to handle.

3 Realization of scattering matrix of the two-dimensional network

Similar to the method proposed by Carlin [2] and Saito [9], the two-dimensional network N mentioned above can be obtained by realizing its $s_{11}(\lambda_1, \lambda_2)$ in terms of the following theorem.

Theorem 2: Given a rational input reflection factor $s_{11}(\lambda_1, \lambda_2)$, ($Z_0$ normalization) satisfying either the lossless or *bounded real* (BR) conditions. Then if $s_{11}(\lambda_1, 1)$ =constant independent of λ_1, a UE related to λ_2 can be extracted whose characteristic impedance is given by

$$Z_{0,t} = Z_0 \frac{1+s_{11}(\lambda_1,1)}{1-s_{11}(\lambda_1,1)}. \quad (15)$$

The rational input reflection factor of the remainder network

$$s_1(\lambda_1,\lambda_2) = \frac{s_{11}(\lambda_1,\lambda_2) - s_{11}(\lambda_1,1)}{1 - s_{11}(\lambda_1,\lambda_2)s_{11}(\lambda_1,1)} \times \frac{1+\lambda_2}{1-\lambda_2} \quad (17)$$

is also lossless (or BR if originally so). Otherwise, one can repeat above procedure with respect to λ_1 for the case (b) or extract a short- or open-circuited stub (related to λ_1) for the cases (a) or (c).

To clearly demonstrate the theorem, several examples are given as below.

Example 1: Assume that the network N has two UEs and its $f(\lambda_1, \lambda_2)$ is expressed as (6). If h_2 and h_4 are chosen to be 0.75 and -0.4167, respectively, the unit normalized scattering parameter $s_{11}(\lambda_1, \lambda_2)$ of the network N can then be obtained by means of (7) and (8) as

$$s_{11}(\lambda_1,\lambda_2) = \frac{0.75\lambda_1 + 1.3333\lambda_2 - 0.4167\lambda_1\lambda_2}{1 + 1.25\lambda_1 + 1.6667\lambda_2 + 1.0833\lambda_1\lambda_2} \quad (18a)$$

or

$$s_{11}(\lambda_1,\lambda_2) = \frac{0.75\lambda_1 + 0.2917\lambda_2 - 0.4167\lambda_1\lambda_2}{1 + 1.25\lambda_1 + 1.0417\lambda_2 + 1.0833\lambda_1\lambda_2}. \quad (18b)$$

For (18a), since $s_{11}(\lambda_1, \lambda_2)|_{\lambda_1=1} = 1/3$, one knows that a UE related to λ_1 can be realized. The characteristic impedance of the UE is calculated by

$$Z_{0,t1} = Z_0 \frac{1+s_{11}(\lambda_1,\lambda_2)}{1-s_{11}(\lambda_1,\lambda_2)}|_{\lambda_1=1} = 2,$$

where $Z_0 = 1$ because of unit normalization. After extracting the cascading UE out of the network N, the input reflection factor of the remainder network normalized with respect to $Z_{0,t1}$ can be achieved in accordance with (17) as

$$s_1(\lambda_2) = \frac{-1 + 2.3333\lambda_2}{3 + 3.6667\lambda_2}.$$

Moreover, another UE related to λ_2 can be easily realized by the similar procedure. The value of its characteristic impedance $Z_{0,t2}$ is 3. Finally, the input reflection factor, s_2, of the remainder network normalized to $Z_{0,t2}$ is known ($s_2 = -0.5$) and supposed unit normalized load resistance R_L can be calculated via

$$R_L = Z_{0,t2}\frac{1+s_2}{1-s_2} = 1.$$

This proves that the numerical calculation completed above is correct. Because the scattering parameter s_{11} is unit normalized, the value of the load resistance R_L terminated on the output port of the network N should be 1.

Similarly, the network N can be synthesized by realizing s_{11} of (18b). However, the UE first extracted is related to λ_2 rather than λ_1. The networks thus synthesized are shown in Fig.1 (a) and (b).

Example 2: Let's assume that the network N consists of a UE and a low-pass element. Then, according to the case (a) described in about section, we may obtain the s_{11} of the network N from the specified h_i as

$$s_{11}(\lambda_1,\lambda_2) = \frac{3\lambda_1 + 0.4167\lambda_2 - 2\lambda_1\lambda_2}{1 + 3\lambda_1 + 1.0833\lambda_2 + 2\lambda_1\lambda_2}. \qquad (19a)$$

In this example, h_2 and h_4 are first assumed to be 3 and -2, respectively, whereas $h_1 = 0$ and $h_3 = 0.4167$ by (5). First of all, we let λ_1 approach to infinity and see if $s_{11}(\infty, \lambda_2)$ is equal to ± 1 or not, because $+1$ corresponds to a series short-circuited stub (or inductance) and -1 to a shunt open-circuited stub (or capacitance). Nevertheless, it can be easily verified that the first element should be UE with its characteristic impedance

$$Z_{0,t1} = Z_0 \frac{1 + s_{11}(\lambda_1, 1)}{1 - s_{11}(\lambda_1, 1)} = \frac{1 + 0.2}{1 - 0.2} = 1.5.$$

Then, the s_1 of the remainder network normalized to $Z_{0,t1}$ can be obtained as

$$s_1(\lambda_1) = \frac{-1 + 12\lambda_1}{5 + 12\lambda_1}$$

It can be seen by letting λ_1 approach to infinity that the next element is a series short-circuited stub (or inductance). Its value can be easily achieved by calculating the corresponding input impedance from s_1, via

$$Z_1(\lambda_1) = Z_{0,t1} \frac{1 + s_1(\lambda_1)}{1 - s_1(\lambda_1)} = 6\lambda_1 + 1.$$

Thus, $Z_{0,sh2}$, the characteristic impedance of the series short-circuited stub is equal to 6. Finally, it is verified from $R_L = 1$ that above numerical computation is correct!

If h_2 and h_4 are assumed to be -2 and -3, respectively, one similarly has

$$s_{11}(\lambda_1,\lambda_2) = \frac{-2\lambda_1 + 0.4167\lambda_2 - 3\lambda_1\lambda_2}{1 + 2\lambda_1 + 1.0833\lambda_2 + 3\lambda_1\lambda_2}. \qquad (19b)$$

By $\lambda_1 \to \infty$, we find that $s_{11}(\infty, \lambda_2) = -1$. It implies that one can first take a shunt open-circuited stub (or capacitance) out. Similar to the normal synthesis procedure, $Z_{0,op1}$, the characteristic impedance of the shunt open-circuited stub is obtained and equal to 0.25. Then, a UE with characteristic impedance $Z_{0,t2}(= 1.5)$ can be extracted. The two networks thus obtained are represented by Fig.2 (a) and (b), respectively.

Example 3: If the network N is assumed to comprise a UE and a high-pass element. Then, in accordance with the case (c) we have

$$s_{11}(\lambda_1,\lambda_2) = \frac{2 + 2.5\lambda_2 - 0.225\lambda_1\lambda_2}{2 + \lambda_1 + 2.5\lambda_2 + 1.025\lambda_1\lambda_2} \qquad (21a)$$

and

$$s_{11}(\lambda_1,\lambda_2) = \frac{2.5 - 2\lambda_2 + 0.225\lambda_1\lambda_2}{2.5 + \lambda_1 + 2\lambda_2 + 1.025\lambda_1\lambda_2} \qquad (21b)$$

under the assumpation of $h_1 = 2$ and $h_3 = 2.5$ for the former equation and $h_1 = 2.5$ and $h_3 = -2$ for the later one, respectively.

When $\lambda_1 = 0$, $s_{11}(\lambda_1, \lambda_2)$ of (21a) is equal to 1. It implies that a series open-circuited stub (or capacitance) should be first extracted whose characteristic impedance $Z_{0,op1}$ equals to 4. In consequence, a UE related to λ_2 can then be obtained. Its $Z_{0,t2}(= 0.8)$ is similarly calculated by the procedure as mentioned above.

On the other hand, it is found that a UE related to λ_2 should be first extracted when realizing $s_{11}(\lambda_1, \lambda_2)$ of (21b). After extracting the UE with characteristic impedance $Z_{0,t1} = 1.25$, a series open-circuited stub (or capacitance) can be easily extracted, whereas the corresponding characteristic impedance $Z_{0,op2} = 0.5$. The networks thus realized with respect to $s_{11}(\lambda_1, \lambda_2)$ of (21a) and (21b) are shown in Fig.3 (a) and (b), respectively.

It should be emphasized that with different choices of h_i, four different topologies can be synthesized for the cases (a) and (c) and two different topologies for the case (b). As to the case (d), it is apparent that there is a one-to-one correspondance between a one-variable network M and two-variable network N. Though Rhodes and Marston [8] had already discussed certain problems associated with the synthesis of the two-variable network N, our method is quite straightforward and simple by using the LTT.

4 Synthesis of multidimensional network constituted by cascaded two-dimensional networks

We now consider the cascade connection of n lossless two-dimensional networks $N_k(k = 1, 2, \cdots, n)$ and denote by f_k, h_k, g_k the corresponding polynomials of the scattering parameters $s_{ij,k}$ of N_k as represented by (1). By calculating the scattering parameters, $s_{ij}^{(k)}$, of the network $N^{(k)}$ constructed by k networks $N_l(l = 1, 2, \cdots, k)$ in cascade as shown by Fig.4, one obtains recurrence formulas for the corresponding polynomials of $s_{ij}^{(k)}$

$$f^{(k)} = f^{(k-1)} f_k, \qquad (22a)$$

$$h^{(k)} = [f^{(k-1)}/(f^{(k-1)})^*](g^{(k-1)})^* h_k + h^{(k-1)} g_k, \qquad (22b)$$

$$g^{(k)} = [f^{(k-1)}/(f^{(k-1)})^*](h^{(k-1)})^* h_k + g^{(k-1)} g_k, \qquad (22c)$$

$$f^{(1)} = f_1, \quad h^{(1)} = h_1, \quad g^{(1)} = g_1, \qquad (22d)$$

$$(k = 2, 3, \cdots, n)$$

where f_k, h_k, and g_k can be determined in accordance with the formulas given in section 2 for one of the four cases. In such a case, the scattering parameter, s_{ij}, of the cascaded 2-ports can be achieved by simply replacing f, h, and g of (1) by $f^{(n)}, h^{(n)}$ and $g^{(n)}$ of (22), respectively.

As an example, a lossless incommensurate line network N inserted between complex generator and load impedances, i.e. Z_G and Z_L is synthesized. Its *transducer power gain* (TPG) can be computed by [7]

$$T = \frac{(1 - |\Gamma_G|^2)|s_{21}|^2(1 - |\Gamma_L|^2)}{|1 - \Gamma_G s_{11}|^2 |1 - \Gamma_2 \Gamma_L|^2}, \qquad (23)$$

where Γ_G and Γ_L are the unit normalized reflection coefficients of the generator and load. They are related to Z_G and Z_L by

$$\Gamma_G = \frac{Z_G - 1}{Z_G + 1}, \quad \Gamma_L = \frac{Z_L - 1}{Z_L + 1} \qquad (24)$$

respectively. Γ_2 is given by

$$\Gamma_2 = s_{22} + \frac{s_{12}s_{21}\Gamma_G}{1 - s_{11}\Gamma_G} \qquad (25)$$

with $s_{ij}(i, j = 1, 2)$ being the entries of the scattering matrix S of the network N.

The network N is composed of two cascaded subnetworks N_1 and N_2. Each of them is assumed to have a UE and a low-pass element as described in the case (a). By optimizing T of (23), their unit normalized $s_{11,k}(k = 1, 2)$ can then obtained as

$$s_{11,1} = \frac{0.512\lambda_1 - 1.4818\lambda_2 - 1.674\lambda_1\lambda_2}{1 + 0.512\lambda_1 + 1.7877\lambda_2 + 1.674\lambda_1\lambda_2}$$

and

$$s_{11,2} = \frac{0.0493\lambda_3 - 3.07747\lambda_4 + 0.3115\lambda_3\lambda_4}{1 + 0.04934\lambda_3 + 3.23587\lambda_4 + 0.3115\lambda_3\lambda_4},$$

Note that λ_i, as the Richards' variables are represented as

$$\lambda_i = tanh(j\omega\tau_i), \qquad (26)$$

where τ_i are the delay lengths of lossless TEM lines (i.e. UEs) and ω the normalized angular frequency.

By analogy with the example 2, the topologies of the subnetworks N_1 and N_2 (see Fig.5(a)) can be obtained by realizing their $s_{11,1}$ and $s_{11,2}$, respectively.

From Fig.5(a) one can see that the series short-circuited stubs $Z_{0,sh2}$ and $Z_{0,sh3}$ are in series with each other and their delay lengths (τ_2 and τ_3) are approximately identical. Therefore, they can be combined as one series short-circuited stub as represented by Fig.5(b). The TPG of Fig.5(b) is shown in Fig.6. It is evident in comparison with the performance of example 1B given in [10] that the TPG of the incommensurate line network thus designed is better than that of lumped reactance network in both pass ($0 \le \omega \le 1$) and stop ($1 < \omega \le 1.4$) regions. Furthermore, no ideal transformer is needed in our example.

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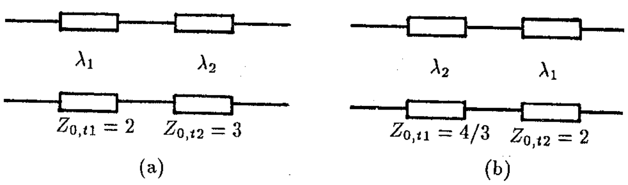

Figure 1: The networks comprising two UEs

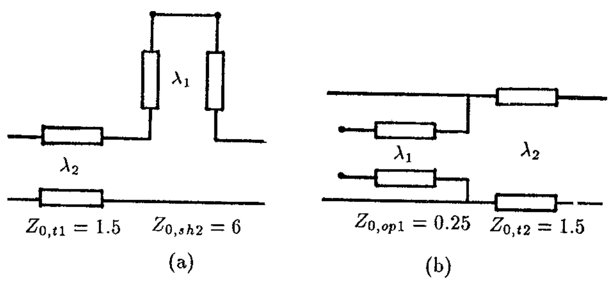

Figure 2: The networks consisting of a UE and a low-pass element

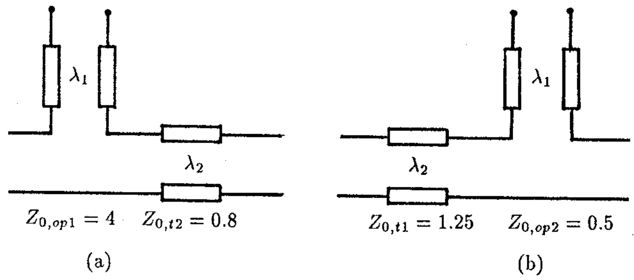

Figure 3: The networks having a UE and a high-pass element

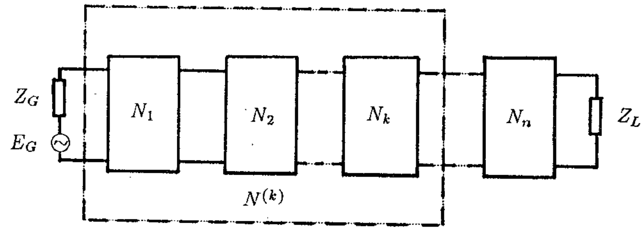

Figure 4: The cascade connection of n lossless two-variable networks $N_k (k = 1, 2, \cdots, n)$

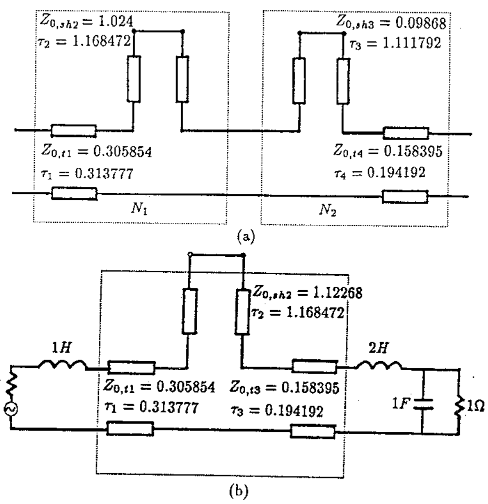

Fig.5 (a) The topologies of subnetworks N_1 and N_2
(b) Incommensurate line network design for example in section 4

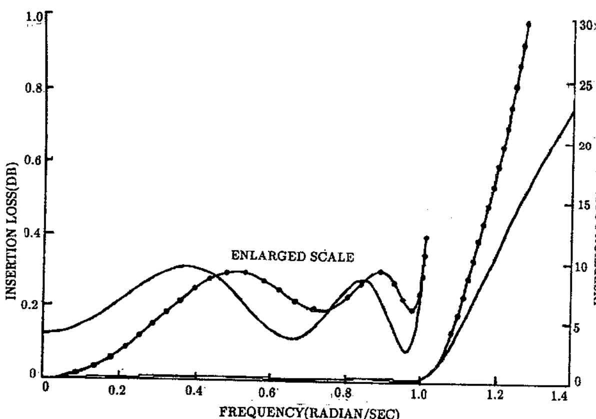

Fig.6 Incommensurate line network and its insertion loss (•-•) comparied with that of [10] (—)